Contracts and Incentives
3. Informativeness and Multitasking

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Informativeness

- In the basic (2x2) moral hazard model, effort increased the likelihood of high output.
- As a consequence, high output served as a signal for high effort and low output served as a signal for low effort.
- With three or more possible output levels it is not clear which output level is a good (the best) indicator of high effort.
- Which output levels should be rewarded? Under what condition should higher outputs receive higher rewards?

Example (1/4)

- Consider three possible output levels with probabilities:

$$\begin{aligned} e = 0: \qquad & \pi_{10} = \frac{1}{12}, \quad \pi_{20} = \frac{5}{12}, \quad \pi_{30} = \frac{6}{12} \\ e = 1: \qquad & \pi_{11} = \frac{1}{12}, \quad \pi_{21} = \frac{3}{12}, \quad \pi_{31} = \frac{8}{12} \end{aligned}$$

- Effort increases the likelihood of $q = 3$ but decreases the likelihood of $q = 2$. The likelihood of $q = 0$ is independent of effort.

Example (2/4)

- When $q = 1$ is observed then $e = 1$ is equally likely than $e = 0$.
- When $q = 2$ is observed then $e = 1$ is less likely than $e = 0$:

$$Prob(e = 1|q = 2) = \frac{\pi_{21}}{\pi_{21} + \pi_{20}} = \frac{3}{8}$$
$$< \frac{5}{8} = \frac{\pi_{20}}{\pi_{21} + \pi_{20}} = Prob(e = 0|q = 2)$$

- When $q = 3$ is observed then $e = 1$ is more likely than $e = 0$:

$$Prob(e = 1|q = 1) = \frac{\pi_{11}}{\pi_{11} + \pi_{10}} = \frac{4}{7}$$
$$> \frac{3}{7} = \frac{\pi_{10}}{\pi_{11} + \pi_{10}} = Prob(e = 0|q = 1)$$

Example (3/4)

- Hence $q = 3$ is a positive signals for effort (and should be rewarded) whereas $q = 2$ is a negative signal (and should be punished).
- The signal $q = 1$ is not informative about effort.
- The optimal payment structure will have U–shape, i.e. it fails to be monotone.
- Non–monotone incentive schemes are hardly ever observed.
- Reason: The agent may hide output in order to increase his payment.

Likelihood Ratios

- Reconsider the FOC of the principal's second best contract:

$$\frac{1}{u'(t_i)} = \mu + \lambda \frac{\pi_{i1} - \pi_{i0}}{\pi_{i1}} \qquad i = 1, 2, 3. \qquad (u'' < 0!)$$

- The optimal payment t_i in output state i is determined by the Likelihood Ratio:

$$L_i \equiv \frac{\pi_{i1} - \pi_{i0}}{\pi_{i1}}$$

- L_i is the relative change of the likelihood of outcome i due to an increase in effort from $e = 0$ to $e = 1$.

Monotone Incentive Schemes

- A sufficient condition for the second best incentive scheme to be monotone is the Monotone Likelihood Ratio Property:

$$L_1 \leq L_2 \leq L_3$$

- Intuition: Output levels with higher likelihood ratio are better indicators of high effort:

$$\frac{Prob(e = 1|q = i)}{Prob(e = 0|q = i)} = \frac{\pi_{i1}}{\pi_{i0}} = 1 - \frac{1}{L_i}$$

First Order Stochastic Dominance

- In the model with two output levels, the principal's expected valuation $V_i = \pi_i v_H + (1 - \pi_i) v_L$ is increasing in effort iff $\pi_1 > \pi_0$.
- For more than two levels things are more complicated.
- A condition that guarantees $V_1 \geq V_0$ is that π_{i1} first order stochastically dominates π_{i0}:

$$Pr(q \leq \tilde{q} | e = 1) \leq Pr(q \leq \tilde{q} | e = 0) \quad \forall \tilde{q} \in [q_{min}, q_{max}]$$

- FOSD means that the cumulative distribution of output is shifted towards higher outputs.

Example (4/4)

- Does FOSD hold in the example?

$$\begin{aligned} e = 0: &\qquad \pi_{10} = \frac{1}{12}, \quad \pi_{20} = \frac{5}{12}, \quad \pi_{30} = \frac{6}{12} \\ e = 1: &\qquad \pi_{11} = \frac{1}{12}, \quad \pi_{21} = \frac{3}{12}, \quad \pi_{31} = \frac{8}{12} \end{aligned}$$

- The example satisfies FOSD since:

$$\begin{aligned} Pr(q \leq 1 | e = 1) = \frac{1}{12} &\quad \leq \quad \frac{1}{12} = Pr(q \leq 1 | e = 0) \\ Pr(q \leq 2 | e = 1) = \frac{4}{12} &\quad \leq \quad \frac{6}{12} = Pr(q \leq 2 | e = 0) \\ Pr(q \leq 3 | e = 1) = \frac{12}{12} &\quad \leq \quad \frac{12}{12} = Pr(q \leq 3 | e = 0) \end{aligned}$$

The Relationship between FOSD and MLRP

- The example satisfies FOSD but MLRP is violated since

$$\frac{\pi_{11} - \pi_{10}}{\pi_{11}} = 0 \not\leq -\frac{2}{3} = \frac{\pi_{21} - \pi_{20}}{\pi_{21}}.$$

- Hence FOSD does not imply MLRP!
- This means that there are situations in which the expected value of production is increasing in effort but the optimal incentive scheme is non–monotone.

Various Sources of Information

- Next to the observation of output, the principal may receive a signal that is informative about the agent's effort choice.
- For example, the agent may be evaluated by a superior.
- Suppose the signal can take two values $s \in \{0, 1\}$.
- When $e = 1$ then $s = 1$ with probability ν_1. When $e = 0$ then $s = 1$ with probability $\nu_0 \leq \nu_1$.
- Definition: The signal is informative iff $\nu_1 > \nu_0$.
- Should the agent's compensation be made contingent on the signal or should it only depend on output?

The Second Best Contract with Signals

- Now there are four outcomes $\{(q_L, 0), (q_H, 0), (q_L, 1), (q_H, 1)\}$ and the principal can choose four payments $t_{L0}, t_{H0}, t_{L1}, t_{H1}$.
- Analog to before the second best compensation schedule solves the FOCs:

$$
\begin{aligned}
\frac{1}{u'(t_{H1})} &= \mu + \lambda \frac{\pi_1 \nu_1 - \pi_0 \nu_0}{\pi_1 \nu_1} \\
\frac{1}{u'(t_{H0})} &= \mu + \lambda \frac{\pi_1 (1 - \nu_1) - \pi_0 (1 - \nu_0)}{\pi_1 (1 - \nu_1)} \\
\frac{1}{u'(t_{L1})} &= \mu + \lambda \frac{(1 - \pi_1) \nu_1 - (1 - \pi_0) \nu_0}{(1 - \pi_1) \nu_1} \\
\frac{1}{u'(t_{L0})} &= \mu + \lambda \frac{(1 - \pi_1)(1 - \nu_1) - (1 - \pi_0)(1 - \nu_0)}{(1 - \pi_1)(1 - \nu_1)}
\end{aligned}
$$

The Informativeness Principle

- Note that $t_{H1} = t_{H0}$ and $t_{L1} = t_{L0}$ only if $\nu_1 = \nu_0$, i.e. when the signal is informative payments should depend on it.
- The Informativeness Principle: Any signal that is informative about an agent's effort should be used to condition the agent's compensation scheme.
- In multi–agent settings this result implies that an agent's compensation should depend on another agent's performance when performances are correlated.

Multitasking

- In the basic moral hazard model the agent performs a single task.
- In most applications agents divide their time between several tasks.
  - Managers take decisions about production, determine the overall strategy of the firm, hire and train employees, etc.
  - Farmers fertilize the land, build fences, feed the cattle, etc.
- How does the provision of incentives for one task affect the effort spent on another?

The Agent

- The agent chooses efforts $e_1, e_2 \geq 0$ for two tasks.
- Effort Costs:

$$c(e_1, e_2) = \frac{1}{2}c_1 e_1^2 + \frac{1}{2}c_2 e_2^2 + \delta e_1 e_2, \qquad 0 < \delta < \sqrt{c_1 c_2}.$$

  For $\delta \to 0$ tasks are independent whereas for $\delta \to 1$ tasks are perfect substitutes.
- Effort Substitution Problem: Raising effort on one task increases the marginal cost of effort on the other task.
- The agent receives a transfer t from the principal and has CARA utility:

$$U(t, e_1, e_2) = -\exp(-r(t - c(e_1, e_2))).$$

Production and Contracts

- Production in task $i \in \{1, 2\}$ is stochastic:

$$q_i = e_i + \epsilon_i.$$

- The random components ϵ_i are normally distributed with zero mean and variance σ_i^2.
- Production in task i contains no information about effort in task j: ϵ_1 and ϵ_2 are independently distributed.
- The agent's transfer is based on (observable) outputs and is assumed to be linear:

$$t = T + t_1 q_1 + t_2 q_2.$$

The Agent's Effort Choice

- The agent chooses e_1 and e_2 to maximize expected utility:

$$E[U] = E\left[-\exp(-r(T + t_1(e_1 + \epsilon_1) + t_2(e_2 + \epsilon_2) - c(e_1, e_2)))\right]$$

- Using $E[\exp(x\epsilon_i)] = \exp(\frac{x^2\sigma_i^2}{2})$:

$$E[U] = -\exp(-r(T + t_1 e_1 + t_2 e_2 - \frac{r}{2}t_1^2\sigma_1^2 - \frac{r}{2}t_2^2\sigma_2^2 - c(e_1, e_2)))$$

- Maximizing expected utility is equivalent to maximizing the Certainty Equivalent:

$$\hat{U}(e_1, e_2) = T + t_1 e_1 + t_2 e_2 - \frac{r}{2}t_1^2\sigma_1^2 - \frac{r}{2}t_2^2\sigma_2^2 - c(e_1, e_2)$$

- Optimal efforts equate marginal benefits with marginal costs:

$$\text{FOC}: \quad t_i = c_i e_i + \delta e_j$$

The Agent's Optimal Effort - Comparative Statics

- Optimal Effort Choice:

$$e_i = \frac{t_i c_j - \delta t_j}{c_i c_j - \delta^2}.$$

- When task j becomes harder to perform (higher c_j), the agent will decrease effort on task j.
- Giving stronger incentive for task i (by raising t_i) leads to higher effort on task i but lower effort on task j.
- The dependence of one task's effort on the other task's incentive pay is increasing in the substitutability of tasks.

The Principal's Problem

- The principal's problem:

$$\max_{T,t_1,t_2} (1-t_1)e_1 + (1-t_2)e_2 - T \qquad \text{s.t.}$$

$$e_1 = \frac{t_1 c_2 - \delta t_2}{c_1 c_2 - \delta^2}, \quad e_2 = \frac{t_2 c_1 - \delta t_1}{c_2 c_1 - \delta^2} \qquad \text{(IC)}$$

$$T + t_1 e_1 + t_2 e_2 - \frac{r}{2} t_1^2 \sigma_1^2 - \frac{r}{2} t_2^2 \sigma_2^2 - c(e_1, e_2) \geq u_R \qquad \text{(PC)}$$

- At the optimum the PC is binding. Substitution leads:

$$\max_{t_1,t_2} \frac{t_1 c_2 - \delta t_2}{c_1 c_2 - \delta^2} + \frac{t_2 c_1 - \delta t_1}{c_2 c_1 - \delta^2} - c\left(\frac{t_1 c_2 - \delta t_2}{c_1 c_2 - \delta^2}, \frac{t_2 c_1 - \delta t_1}{c_2 c_1 - \delta^2}\right)$$
$$- u_R - \frac{r}{2} t_1^2 \sigma_1^2 - \frac{r}{2} t_2^2 \sigma_2^2$$

The Second Best Contract

- First order condition:

$$t_i = \frac{c_j - \delta(1 - t_j)}{c_j + r\sigma_i^2(c_i c_j - \delta^2)}.$$

- When tasks are independent ($\delta \to 0$) then second best incentives are determined by the standard tradeoff between risk sharing and incentives:

$$t_i = \frac{1}{1 + r\sigma_i^2 c_i}.$$

Complementarity of Incentives

- When tasks are substitutes then second best incentives are complementary:

$$t_j \uparrow \Rightarrow t_i \uparrow .$$

- The solution:

$$t_i^{**} = \frac{1 + (c_j - \delta) r \sigma_j^2}{1 + r(c_i \sigma_i^2 + c_j \sigma_j^2) + r^2 \sigma_i^2 \sigma_j^2 (c_i c_j - \delta^2)}$$

Low Powered Incentives

- Suppose that σ_j^2 increases, so that task j becomes less measurable. Due to the tradeoff between risk sharing and incentives t_j^{**} has to decrease. Since incentives are complementary t_i^{**} has to decrease as well.
- Taking $\sigma_j^2 \to \infty$ this logic implies that if the principal cares only about a task whose output is unobservable, he should give *no incentives at all*.
- Example: Teachers' pay should not be linked to test scores if we care about the teaching of (social) skills that cannot be measured in such tests.

Conflicting Tasks and Job Design

- In many examples there is a direct conflict between tasks.
- Example: A manufacturer employs a salesman to sell two products which are imperfect substitutes.
- The sales of product i increase in the promotion effort for product i but decrease in the promotion effort for product j.
- How to provide incentives for conflicting tasks? And should conflicting tasks be executed by separate agents?

A 2x2 Model with Conflicting Tasks

- A sales agent chooses promotion efforts $e_i \in \{0, 1\}$ at a unit cost c for two products $i = 1, 2$ sold at price 1.
- The likelihood of selling product i is given by

$$Pr(q_i = 1) = \alpha + \rho e_i - \gamma e_j.$$

- Assume that promoting both products is efficient in that

$$\rho - \gamma > c.$$

- The agent is risk neutral and payments cannot be negative.

The Principal's Problem

- By symmetry the principal's transfer is $t(q_i, q_j) \in \{t_0, t_1, t_2\}$ and limited liability implies $t_0 = 0$.
- Defining $\phi = \alpha + \rho - \gamma$ the principal's problem is:

$$\min_{t_1, t_2 \geq 0} \phi^2 t_2 + 2\phi(1-\phi)t_1 \quad \text{s.t.}$$

  - IC(M):

$$\phi^2 t_2 + 2\phi(1-\phi)t_1 - 2c \geq (\alpha+\rho)(\alpha-\gamma)t_2$$
$$+[(\alpha+\rho)(1-\alpha+\gamma) + (\alpha-\gamma)(1-\alpha-\rho)]t_1 - c$$

  - IC(L):

$$\phi^2 t_2 + 2\phi(1-\phi)t_1 - 2c \geq \alpha^2 t_2 + 2\alpha(1-\alpha)t_1$$

Solution: No Reward for Intermediate Performance

- Suppose that $t_1 > 0$. Decrease t_1 by $\phi \Delta t$ units and increase t_2 by $2(1-\phi)\Delta t$ units.
- This leaves LHS of ICs unchanged:

$$\phi^2 \cdot 2(1-\phi)\Delta t - 2\phi(1-\phi) \cdot \phi \Delta t = 0.$$

- The RHS of the ICs decrease since t_2 is less likely to be obtained when effort is lower:

$$\alpha^2 \cdot 2(1-\phi)\Delta t - 2\alpha(1-\alpha) \cdot \phi \Delta t < 0.$$

- Since IC are relaxed, expected compensation can be lowered until $t_1^{**} = 0$ in the second best.

Solution: IC(L) implies IC(M)

- Given $t_1^{**} = 0$ the IC constraints become:

$$\begin{aligned} \phi^2 t_2 - 2c &\geq (\alpha+\rho)(\alpha-\gamma)t_2 - c \\ \phi^2 t_2 - 2c &\geq \alpha^2 t_2. \end{aligned}$$

- The benefit of effort has to exceed its costs:

$$\begin{aligned} t_2[\phi^2 - (\alpha+\rho)(\alpha-\gamma)] &\geq c \\ t_2[\phi^2 - \alpha^2] &\geq 2c \end{aligned}$$

- If two units of effort are preferred over zero units, i.e. if $t_2 \geq \frac{2c}{\phi^2-\alpha^2}$ then

$$t_2[\phi^2 - (\alpha+\rho)(\alpha-\gamma)] \geq \frac{2[\phi^2 - (\alpha+\rho)(\alpha-\gamma)]}{\phi^2-\alpha^2} c > c$$

i.e. two units of effort are preferred over one unit.

The Second Best Contract under Multitasking

- At the optimum IC(L) has to be binding:

$$t_2^{**} = \frac{2c}{\phi^2 - \alpha^2}.$$

- The agent obtains a rent:

$$U = \phi^2 t_2^{**} - 2c = 2c\frac{\alpha^2}{\phi^2 - \alpha^2}$$

- The principal's cost of inducing promotion efforts for both products is

$$C^{SB} = \phi^2 t_2^{**}.$$

- Can the principal lower these costs by having the two products promoted by two separate agents?

Separation of Tasks

- The principal hires one agent for each task. Symmetry implies that both agents receive the same contract.
- Agent i receives a transfer only if his product is sold, i.e. $t_0 = 0$.
- Agent i's transfer may depend on whether agent j has sold his product or not, i.e. $t \in \{t_1, t_2\}$.
- For any pair of transfers t_1, t_2 agents will play a Nash equilibrium in effort choices.

The Principal's Problem

- The principal's problem:

$$\min_{t_1,t_2} \phi^2 t_2 + \phi(1-\phi)t_1 \quad \text{s.t.}$$

$$\phi[\phi t_2 + (1-\phi)t_1] - c \geq (\phi-\rho)[(\phi+\gamma)t_2 + (1-\phi-\gamma)t_1] \quad \text{(NE)}$$

- Exerting effort raises the agent's own probability of success and lowers the rival's.
- It is optimal to set $t_2^{**} = 0$: Agent i has to be punished (maximally) for agent j's success because it indicates that agent i has exerted low effort.

The Second Best Contract under Separation of Tasks

- Setting $t_2^{**} = 0$ in (NE) implies

$$t_1^{**} = \frac{c}{\phi(1-\phi) - (\phi-\rho)(1-\phi-\gamma)}.$$

- Each agent receives a rent:

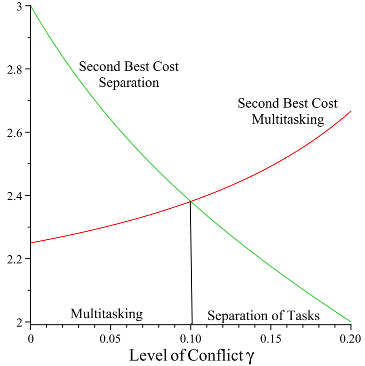
$$U = \phi(1-\phi)t_1^{**} - c = \frac{(\phi-\rho)(1-\phi-\gamma)}{\phi(1-\phi) - (\phi-\rho)(1-\phi-\gamma)}c$$

- The cost of inducing promotion efforts for both products is

$$C^{SB} = 2\phi(1-\phi)t_1^{**}$$

Job Design

- The principal will employ two separate agents when the costs of inducing effort are lower than for a single agent.
- Example: $\alpha = 0.2$, $\rho = 0.4$



- The optimal job design gives an agent responsibility over multiple tasks as long as the level of conflict between tasks is sufficiently low.

Multitasking vs Singletasking

- The advantage of multitasking is that the agent can coordinate his efforts. What matters is the choice between effort on both tasks or no effort at all.
- The advantage of single tasking is that agents take rents from each other by exerting effort. An increase in one agent's effort lowers the probability with which the principal needs to reward the other.

Application - Pure Conflict

- Suppose a principal hires one or two agents to search for information about the pros and cons of a decision.
- Agents cannot be rewarded based on the amount of information obtained but only on the basis of the decision's outcome.
- When the decision has two outcomes (yes/no) then there is pure conflict and two agents should be employed.
- The agent searching for positive (negative) information should be rewarded when the decision is "yes" ("no").
- Example: Prosecution and Defense in judicial system.

Summary

- Incentive payments should be monotone increasing in output when higher outputs are more informative about high effort than lower outputs (MLRP).
- MLRP implies FOSD but FOSD does not imply MLRP.
- Any signal that is informative about an agent's effort should be used to condition the agent's compensation scheme.
- When an agent works on multiple tasks and efforts are substitutes in the agent's cost function then second best incentives are complementary across tasks.
- Conflicting tasks should be assigned to separate agents if and only if the level of conflict is sufficiently strong.

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